Using Blinder-Oaxaca Decomposition to Explore Differential Item Functioning: Application to PISA 2009 Reading

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Overview

- Manifest versus Latent Differential Item Functioning
- Overview of Blinder-Oaxaca Decomposition
- Extending Blinder-Oaxaca to Categorical Outcomes
- Application to Differential Item Functioning Analyses
  - Simulation Illustrations
  - PISA 2009: Gender DIF in Reading, Canada

Differential Item Functioning

- The manifest person characteristics considered when evaluating test fairness (e.g., gender, nationality) may be only weakly related to variables most responsible for DIF
- Items that display DIF for a particular manifest population may do so for very different reasons and in relation to different subgroups within that population
- Contemporary assessments (e.g., PISA) often collect many additional variables that can be explored in relation to DIF

Blinder-Oaxaca Decomposition

Consider groups $g = (A, B)$, an outcome variable Y_{ig}, a predictor variable matrix $\mathbf{X}_{ig}$, and a linear relationship between outcome and predictors given by:

$$Y_{ig} = \mathbf{X}_{ig}\beta_g + \epsilon_{ig}. \tag{1}$$

Blinder (1973) and Oaxaca (1973) suggested a decomposition of the form

$$\bar{Y}_A - \bar{Y}_B = \Delta^{OLS} = (\bar{X}_A - \bar{X}_B)\hat{\beta}_A + \bar{X}_B(\hat{\beta}_A - \hat{\beta}_B), \tag{2}$$

where $\bar{Y}_g = N_g^{-1} \sum_{i=1}^{N_g} Y_{ig}$ and $\bar{X}_g = N_g^{-1} \sum_{i=1}^{N_g} X_{ig}$

Extending Blinder-Oaxaca Decomposition to the Nonlinear Case (Sinning, Hahn & Bauer, 2008)

The Blinder-Oaxaca Decomposition can be written alternatively in the form of conditional expectations:

$$\Delta_A^{NL} = \{E_{\beta_A}(Y_{iA}|\mathbf{X}_{iA}) - E_{\beta_A}(Y_{iB}|\mathbf{X}_{iB})\} + \{E_{\beta_A}(Y_{iB}|\mathbf{X}_{iB}) - E_{\beta_B}(Y_{iB}|\mathbf{X}_{iB})\}$$

or alternatively,

$$\Delta_B^{NL} = \{E_{\beta_B}(Y_{iA}|\mathbf{X}_{iA}) - E_{\beta_B}(Y_{iB}|\mathbf{X}_{iB})\} + \{E_{\beta_A}(Y_{iA}|\mathbf{X}_{iA}) - E_{\beta_B}(Y_{iA}|\mathbf{X}_{iA})\}$$

Group Differences Due to Differences in Endowments (E) versus Coefficients (C)

$$\Delta_A^{NL} = \underbrace{\{E_{\beta_A}(Y_{iA}|\mathbf{X}_{iA}) - E_{\beta_A}(Y_{iB}|\mathbf{X}_{iB})\}}_{\text{E}} + \underbrace{\{E_{\beta_A}(Y_{iB}|\mathbf{X}_{iB}) - E_{\beta_B}(Y_{iB}|\mathbf{X}_{iB})\}}_{\text{C}}$$

E = difference in outcome due to differences in Endowments

C = difference in outcome due to differences in Coefficients

Uniform and Non-Uniform DIF as Potential Sources of Item Score Differences Across Groups

The Blinder-Oaxaca decomposition shares similarities to how group differences are studied in IRT-based (i.e., 2PL) DIF analyses, where overall item score differences are understood as occurring in relation to:

- Group differences in proficiency (i.e., ‘impact”, not DIF)
- Main effects related to group (i.e., differences in the b-parameter, or “uniform DIF”)
- Interactions between group and proficiency (i.e., differences in the a-parameter, or “nonuniform DIF”)

Uniform and Non-Uniform DIF as Potential Sources of Item Score Differences Across Groups

A primary difference with Blinder-Oaxaca is that these sources are evaluated by their relative contributions to mean item score differences, not by their simple presence. For example, significant nonuniform DIF may not contribute to a mean score difference across groups.

Using Blinder-Oaxaca to Inform DIF Analyses

The value of Blinder-Oaxaca in DIF analyses comes from its potential to better understand what contributes to DIF. Person-level covariates can function in different ways in explaining group differences. Evaluating such effects may help in evaluating whether the observation of DIF reflects a lack of validity.

A Simulation Illustration

- We assume two groups (A,B) are studied in relation to an item score Y, and that the individuals in each group have proficiency θ and are measured on four binary covariates X_1, X_2, X_3, X_4.
- $\mathbf{X}_{ig} = (\theta, X_1^*, X_2^*, X_3^*, X_4^*) \sim MVN(\mu_g, \Sigma)$
- $\mu_A = (-.5, .25, -.25, .5, 0); \mu_B = (.5, -.25, .25, -.5, 0)$
- $diag(\Sigma) = (1, 1, 1, 1, 1), \sigma_{ij} = .2$ for all $i \neq j$
- $X_j = I(X_j^* > 0)$
- We further manipulate the effects of the covariates and residual group differences across five items to illustrate how sources of DIF can be studied through the Blinder-Oaxaca decomposition

A Simulation Illustration

We simulate DIF of three types for five different items. In each case we simulate DIF through the *b* parameter of a 2PL model:

- 1. $b = -.75 + .75 * X_3 + .5 * (Group = A)$
- 2. $b = -.75 + .75 * X_3$
- 3. $b = -.75 + .5 * (Group = A)$
- 4. $b = -.75 + 1.0 * X_4 * (Group = A) + .5 * (Group = A)$
- 5. $b = -.75 + .75 * X_3 + .5 * (Group = A) + 1.0 * X_4 * (Group = A)$;

A Simulation Illustration

- We simulate 5000 examinees per group, using the 2PL to generate item responses.
- For all items, $a = 1$.
- We apply the Blinder-Oaxaca decomposition using the STATA package *mvdcmp* (Powers, Yoshioka & Yun, 2011); $\theta, X_1, X_2, X_3, X_4$ are entered as predictors for all three items

Simulation Analysis: Item 1

```
. mvdcmp group: logit item1 theta cov1 cov2 cov3 cov4
```

Decomposition Results — Number of obs = 10000

High outcome group: group==1 --- Low outcome group: group==0

item1	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]	Pct.
E	.24952	.0096046	25.98	0.000	.23069	.26834	70.845
C	.10268	.013291	7.73	0.000	.076633	.12873	29.155
R	.3522	.0086131	40.89	0.000	.33532	.36908	

Due to Difference in Characteristics (E)

item1	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]	Pct.
theta	**.19843**	**.0061258**	**32.39**	**0.000**	**.18643**	**.21044**	**56.341**
cov1	-.0043638	.0030991	-1.41	0.159	-.010438	.0017104	-1.239
cov2	-.00093315	.0026626	-0.35	0.726	-.0061519	.0042856	-.26495
cov3	**.056697**	**.0053856**	**10.53**	**0.000**	**.046141**	**.067253**	**16.098**
cov4	-.00031552	.00029099	-1.08	0.278	-.00088585	.00025482	-.089585

Due to Difference in Coefficients (C)

item1	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]	Pct.
theta	-.0065779	.0058057	-1.13	0.257	-.017957	.0048013	-1.8677
cov1	.016193	.012158	1.33	0.183	-.0076369	.040023	4.5977
cov2	.00023676	.0079536	0.03	0.976	-.015352	.015826	.067225
cov3	-.0081492	.014591	-0.56	0.576	-.036747	.020449	-2.3138
cov4	-.012475	.0094954	-1.31	0.189	-.031086	.0061359	-3.542
_cons	**.11346**	**.020987**	**5.41**	**0.000**	**.072321**	**.15459**	**32.213**

Simulation Analysis: Item 2

```
. mvdcmp group: logit item2 theta cov1 cov2 cov3 cov4

Decomposition Results                                       Number of obs =     10000
-------------------------------------------------------------------------------------
High outcome group: group==1 ---  Low outcome group: group==0
-------------------------------------------------------------------------------------
      Item2|      Coef.  Std. Err.       z     P>|z|     [95% Conf. Interval]    Pct.
-----------+-------------------------------------------------------------------------
          E|      .2522   .0092152    27.37    0.000      .23414      .27026  99.213
          C|   .0020004    .013091     0.15    0.879    -.023658     .027658  .78692
-----------+-------------------------------------------------------------------------
          R|      .2542   .0086686    29.32    0.000      .23721      .27119

                      Due to Difference in Characteristics (E)
-------------------------------------------------------------------------------------
     Item2 |      Coef.  Std. Err.       z     P>|z|     [95% Conf. Interval]    Pct.
-----------+-------------------------------------------------------------------------
     theta |     .19555   .0058825    33.24    0.000      .18402      .20707  76.926
      cov1 |  .00093051   .0027122     0.34    0.732   -.0043854    .0062464  .36605
      cov2 | -.00097095   .0025308    -0.38    0.701   -.0059314    .0039895 -.38196
      cov3 |    .057145   .0054294    10.53    0.000     .046504     .067787   22.48
      cov4 | -.00045037  .00027395    -1.64    0.100  -.00098732  .000086583 -.17717
-------------------------------------------------------------------------------------

                       Due to Difference in Coefficients (C)
-------------------------------------------------------------------------------------
     Item2 |      Coef.  Std. Err.       z     P>|z|     [95% Conf. Interval]    Pct.
-----------+-------------------------------------------------------------------------
     theta |   .0030244   .0073982     0.41    0.683    -.011476     .017525  1.1898
      cov1 |  -.0073224    .013889    -0.53    0.598    -.034545       .0199 -2.8806
      cov2 |  -.0031469   .0084347    -0.37    0.709    -.019679     .013385  -1.238
      cov3 |  -.0040488     .01045    -0.39    0.698     -.02453     .016432 -1.5928
      cov4 |   -.015286    .027718    -0.55    0.581    -.069614     .039042 -6.0133
     _cons |     .02878    .056687     0.51    0.612    -.082326      .13989  11.322
-------------------------------------------------------------------------------------
```

Simulation Analysis: Item 3

```
. mvdcmp group: logit item3 theta cov1 cov2 cov3 cov4

Decomposition Results                                   Number of obs =     10000
-----------------------------------------------------------------------------------
High outcome group: group==1 ---  Low outcome group: group==0
-----------------------------------------------------------------------------------
      Item3|      Coef.  Std. Err.        z    P>|z|    [95% Conf. Interval]    Pct.
-----------+-----------------------------------------------------------------------
          E|      .1558    .010422    14.95    0.000      .13537      .17622  47.441
          C|      .1726    .014026    12.31    0.000      .14511       .2001  52.559
-----------+-----------------------------------------------------------------------
          R|      .3284   .0085827    38.26    0.000      .31158      .34522

                      Due to Difference in Characteristics (E)
-----------------------------------------------------------------------------------
     Item3 |      Coef.  Std. Err.        z    P>|z|    [95% Conf. Interval]    Pct.
-----------+-----------------------------------------------------------------------
     theta |     .16983   .0063525    26.73    0.000      .15738      .18228  51.715
      cov1 |  -.0069561   .0029146    -2.39    0.017     -.012669   -.0012434 -2.1182
      cov2 |   .0020633   .0026084     0.79    0.429    -.0030492    .0071758  .62829
      cov3 |  -.0091608   .0056859    -1.61    0.107     -.020305    .0019835 -2.7895
      cov4 | .000016503 .000045249     0.36    0.715 -.000072185   .00010519 .005025
-----------------------------------------------------------------------------------

                       Due to Difference in Coefficients (C)
-----------------------------------------------------------------------------------
     Item3 |      Coef.  Std. Err.        z    P>|z|    [95% Conf. Interval]    Pct.
-----------+-----------------------------------------------------------------------
     theta |   .0020581   .0056062     0.37    0.714    -.0089301     .013046   .6267
      cov1 |    .019832    .012201     1.63    0.104    -.0040814     .043746   6.039
      cov2 |   .0039845   .0079234     0.50    0.615     -.011545     .019514  1.2133
      cov3 |    .010286     .01522     0.68    0.499     -.019545     .040118  3.1322
      cov4 |   .0015798   .0097462     0.16    0.871     -.017523     .020682  .48107
     _cons |     .13486     .02097     6.43    0.000      .093763      .17597  41.067
```

Simulation Analysis: Item 4

```
. mvdcmp group: logit item4 theta cov1 cov2 cov3 cov4

Decomposition Results                                  Number of obs =     10000
----------------------------------------------------------------------------------
High outcome group: group==1 ---  Low outcome group: group==0
----------------------------------------------------------------------------------
      Item4|      Coef.  Std. Err.       z    P>|z|     [95% Conf. Interval]    Pct.
-----------+----------------------------------------------------------------------
          E|     .18211    .010788   16.88    0.000      .16097      .20325  49.033
          C|     .18929    .014274   13.26    0.000      .16131      .21727  50.967
-----------+----------------------------------------------------------------------
          R|      .3714   .0084675   43.86    0.000       .3548        .388

                      Due to Difference in Characteristics (E)
----------------------------------------------------------------------------------
     Item4 |      Coef.  Std. Err.       z    P>|z|     [95% Conf. Interval]    Pct.
-----------+----------------------------------------------------------------------
     theta |     .18141    .006732   26.95    0.000      .16821       .1946  48.844
      cov1 |  -.0008371   .0030756   -0.27    0.785    -.0068654    .0051912 -.22539
      cov2 |   .0061449   .0026025    2.36    0.018     .0010441     .011246  1.6545
      cov3 |  -.0048524   .0056465   -0.86    0.390     -.015919    .0062147 -1.3065
      cov4 |  .00024916  .00028654    0.87    0.385   -.00031246   .00081078 .067086
----------------------------------------------------------------------------------

                       Due to Difference in Coefficients (C)
----------------------------------------------------------------------------------
     Item4 |      Coef.  Std. Err.       z    P>|z|     [95% Conf. Interval]    Pct.
-----------+----------------------------------------------------------------------
     theta |   .0024804   .0057825    0.43    0.668    -.0088533     .013814  .66784
      cov1 |    .012818    .012346    1.04    0.299     -.011379     .037016  3.4513
      cov2 |    .015114   .0080287    1.88    0.060   -.00062242      .03085  4.0694
      cov3 |  -.0099652    .015052   -0.66    0.508     -.039468     .019537 -2.6831
      cov4 |     .10532   .0095154   11.07    0.000     .086665      .12397  28.356
     _cons |    .063527    .020768    3.06    0.002     .022823      .10423  17.105
----------------------------------------------------------------------------------

.
```

Simulation Analysis: Item 5

```
. mvdcmp group: logit item5 theta cov1 cov2 cov3 cov4

Decomposition Results                                  Number of obs =     10000
----------------------------------------------------------------------------------
High outcome group: group==1 ---  Low outcome group: group==0
----------------------------------------------------------------------------------
      Item5|      Coef.  Std. Err.       z     P>|z|    [95% Conf. Interval]    Pct.
-----------+----------------------------------------------------------------------
          E|     .25862   .0096044   26.93    0.000      .23979      .27744  58.116
          C|     .18638     .01307   14.26    0.000      .16077        .212  41.884
-----------+----------------------------------------------------------------------
          R|       .445   .0081801   54.40    0.000      .42897      .46103

                      Due to Difference in Characteristics (E)
----------------------------------------------------------------------------------
     Item5 |      Coef.  Std. Err.       z     P>|z|    [95% Conf. Interval]    Pct.
-----------+----------------------------------------------------------------------
     theta |     .19695   .0061842   31.85    0.000      .18483      .20907  44.259
      cov1 |   .0016617   .0031077    0.53    0.593   -.0044293    .0077527  .37342
      cov2 |  -.0016266   .0026781   -0.61    0.544   -.0068758    .0036225 -.36553
      cov3 |    .061757   .0053693   11.50    0.000     .051233      .07228  13.878
      cov4 | -.00012733   .0002923   -0.44    0.663  -.00070024   .00044559-.028613
----------------------------------------------------------------------------------

                       Due to Difference in Coefficients (C)
----------------------------------------------------------------------------------
     Item5 |      Coef.  Std. Err.       z     P>|z|    [95% Conf. Interval]    Pct.
-----------+----------------------------------------------------------------------
     theta |   .0039819    .005322    0.75    0.454   -.0064492     .014413   .8948
      cov1 |  -.0077574    .011326   -0.68    0.493    -.029957     .014442 -1.7432
      cov2 |   .0019748   .0074684    0.26    0.791    -.012663     .016613  .44378
      cov3 |  -.0060359    .013504   -0.45    0.655    -.032504     .020432 -1.3564
      cov4 |    .091144   .0086422   10.55    0.000     .074205      .10808  20.482
     _cons |     .10308    .019754    5.22    0.000     .064358      .14179  23.163
----------------------------------------------------------------------------------
```

2009 PISA Reading, Gender Differences

- Chuy & Nitulescu (2013) consider differences across gender groups for PISA 2009 reading scores in Canada
- PISA scores are studied using Blinder-Oaxaca decomposition in relation to (1) reading engagement and (2) approaches to learning variables
- The overall higher performance of females in comparison to males in terms of PISA scores is found to be affected by both sets of variables

2009 PISA Reading, Gender Differences

	Minimum	Maximum	Mean	Standard Deviation
Male	99.64	828.56	507.18	92.35
Female	79.50	843.46	541.53	84.86

*PISA 2009 data for Canada, Calculations by Chuy & Nitulescu (2013)

2009 PISA Reading, Gender Differences

- Approaches to learning variables
  - Cognitive strategies: Memorization, Elaboration, Control
  - Meta-cognitive strategies: Understanding and Remembering, Summarizing
- Engagement-in-reading variables
  - Reading enjoyment
  - On-line reading
  - Reading diversity

2009 PISA Reading, Gender Differences in Cognitive and Meta-cognitive Strategies

	Cognitive strategies			Meta-cognitive strategies	
	Memorize	Elaborate	Control	Understand	Summarize
Male	-.16 (.02)	-.16 (.02)	-.09 (.02)	-.17 (.02)	-.19 (.01)
Female	.12 (.02)	-.25 (.01)	.30 (.01)	.12 (.01)	.24 (.01)
Difference	-.28 (.02)	.09 (.02)	-.39 (.02)	-.29 (.02)	-.43 (.02)

*PISA 2009 data for Canada, Calculations by Chuy & Nitulescu (2013)

2009 PISA Reading, Effects of Cognitive and Meta-cognitive Strategies on PISA Scores, Multiple Regression

Dependent Variable: PISA Reading Score

		Coefficient	Standard Error
Cognitive Strategies	Memorize	-12.40	.83
	Elaborate	-8.56	.86
	Control	28.58	.99
Meta-cognitive strategies	Understand	10.66	.92
	Summarize	23.15	.85

*PISA 2009 data for Canada, Calculations by Chuy & Nitulescu (2013)

2009 PISA Reading, Gender Differences in Engagement-in-Reading Activities

	Reading Enjoyment	On-line Reading	Reading Diversity
Male	-.28 (.02)	-.03 (.02)	-.24 (.02)
Female	.55 (.02)	-.04 (.01)	.01 (.01)
Difference	-.83 (.02)	.00 (.02)	-.25 (.02)

*PISA 2009 data for Canada, Calculations by Chuy & Nitulescu (2013)

2009 PISA Reading, Effects of Engagement-in-Reading Activities on PISA Scores

	Coefficient	Standard Error
Reading Enjoyment	35.70	.80
On-line reading	14.07	1.25
Reading Diversity	18.41	.98

*PISA 2009 data for Canada, Calculations by Chuy & Nitulescu (2013)

PISA Items R452Q06 and R458Q07

R452Q06	Coef.	Std. Err.	z	P>|z|
MALE	-.2591	.0847	-3.06	0.002
PROF	.0155	.0006	24.26	0.000
_cons	-7.6714	.3406	-22.52	0.000

R458Q07	Coef.	Std. Err.	z	P>|z|
MALE	.4988	.0875	5.70	0.000
PROF	.0140	.0006	22.88	0.000
_cons	-6.6301	.3177	-20.87	0.000

PISA Item R452Q06

```
. mvdcmp ST04Q01, reverse: logit R452Q06 PROF METASUM UNDREM MEMOR ELAB CSTRAT JOYREAD ONLNREAD
DIVREAD [pw=W_FSTUWT]
```

Decomposition Results — Number of obs = 23207

High outcome group: ST04Q01==0 --- Low outcome group: ST04Q01==1

R452Q06	Coef.	Std. Err.	z	P>|z|	Pct.
E	.1206	.0095	12.70	0.000	88.73
C	.0153	.0187	.82	0.411	11.27
R	.1359	.01568	8.67	0.000	

Due to Difference in Characteristics (E)

R452Q06	Coef.	Std. Err.	z	P>|z|	Pct.
PROF	**.0983**	**.0050**	**19.84**	**0.000**	**72.32**
METASUM	.0032	.0056	0.54	0.592	2.33
UNDREM	.0002	.0037	0.06	0.955	.15
MEMOR	.0028	.0032	0.87	0.385	2.05
ELAB	.0007	.0011	0.67	0.501	.55
CSTRAT	-.0021	.0059	-0.35	0.724	-1.53
JOYREAD	**.0253**	**.0105**	**2.41**	**0.016**	**18.60**
ONLNREAD	-.0001	.0002	-0.25	0.804	-.04
DIVREAD	-.0005	.0038	-0.12	0.903	-.34

Due to Difference in Coefficients (C)

R452Q06	Coef.	Std. Err.	z	P>|z|	Pct.
PROF	.2334	.1439	1.62	0.105	171.68
METASUM	-.0028	.0037	-0.77	0.441	-2.09
UNDREM	.0012	.0031	0.40	0.692	.90
MEMOR	-.0018	.0029	-0.61	0.542	-1.29
ELAB	.0019	.0030	0.64	0.521	1.42
CSTRAT	.0000	.0015	0.00	0.997	.00
JOYREAD	.0069	.0058	1.20	0.231	5.11
ONLNREAD	-.0002	.0003	-0.69	0.489	-.16
DIVREAD	-.0043	.0047	-0.91	0.365	-3.14
_cons	-.2190	.1475	-1.48	0.138	-161.17

PISA Item R458Q07

```
. mvdcmp ST04Q01, reverse: logit R458Q07 PROF METASUM UNDREM MEMOR ELAB CSTRAT JOYREAD ONLNREAD
DIVREAD [pw=W_FSTUWT]
```

Decomposition Results Number of obs = 23207

High outcome group: ST04Q01==0 --- Low outcome group: ST04Q01==1

R452Q06	Coef.	Std. Err.	z	P>|z|	Pct.
E	.0923	.0095	9.69	0.000	10983.22
C	-.0928	.0180	-5.16	0.000	-10883.22
R	-.0005	.0150	-.04	0.971	

Due to Difference in Characteristics (E)

R452Q06	Coef.	Std. Err.	z	P>|z|	Pct.
PROF	**.0832**	**.0058**	**14.36**	**0.000**	**10131.92**
METASUM	-.0049	.0058	-0.84	0.399	-654.32
UNDREM	.0068	.0037	1.82	0.068	849.04
MEMOR	**-.0074**	**.0038**	**-1.96**	**0.050**	**-845.12**
ELAB	-.0003	.0012	-0.28	0.782	-44.30
CSTRAT	.0051	.0063	0.81	0.420	589.88
JOYREAD	.0122	.0105	1.16	0.248	1175.70
ONLNREAD	.0002	.0003	0.81	0.418	24.12
DIVREAD	.0033	.0041	0.80	0.424	450.03

Due to Difference in Coefficients (C)

R452Q06	Coef.	Std. Err.	z	P>|z|	Pct.
PROF	**-.2551**	**.1260**	**-2.03**	**0.043**	**-28765.14**
METASUM	.0040	.0032	1.26	0.208	531.86
UNDREM	-.0039	.0027	-1.46	0.145	-504.78
MEMOR	**.0061**	**.0026**	**2.30**	**0.021**	**730.96**
ELAB	.0011	.0027	0.39	0.695	139.40
CSTRAT	-.0025	.0014	-1.78	0.076	-304.34
JOYREAD	-.0055	.0048	-1.15	0.252	-573.47
ONLNREAD	.0003	.0003	1.21	0.227	40.71
DIVREAD	-.0028	.0041	-0.69	0.492	-384.05
_cons	.1656	.1277	1.30	0.195	18205.66

Conclusions

- Preliminary results suggest that Oaxaca-Blinder decomposition can be a useful tool in interpreting DIF findings, especially in the presence of many covariates
- PISA reading items appear differentially sensitive to approaches-to-learning and engagement-in-reading variables; such differences appear at least partially responsible for the emergence of DIF
- The analyses may be further informed by attempts to distinguish explanatory variables as construct relevant versus irrelevant.